2 Social Welfare Functions and Social Insurance Contracts

*The choice of a social-insurance contract behind the veil of ignorance – to be implemented through government when people will know who they are – is correspondingly the choice of a social welfare function.*

### A Social Welfare Function

- When we examined efficiency, we looked at outcomes that maximized net social surplus \( W = B - C \)
  - Disregarding the cost side for the moment, we were concerned with the maximization of the sum across consumers of their individual surpluses
    \[
    B = \sum_{i=1}^{n} B_i
    \]
    - Because the coefficient of each \( B_i \) term is unity, this means that we weighted the welfare of individual consumers equally

- There is no reason why this would have to be so: society might value the utility of particular individuals more than that of others
• In the pursuit of social justice and the optimal social-insurance contract, we consider a more general social welfare function $W$
  
  – This function includes general judgements about the distribution of benefits and costs across people in the population
  
  – Thus, it need no longer be the case that individuals are weighed symmetrically

• With $n$ people in society, a social welfare function measures total welfare $W$ of the population as:

\[ W = f(U_1, U_2, \cdots, U_n) \]  

– Total social welfare $W$ thus depends on the values of individual utilities $U_i$ in the population

– Because social welfare as expressed here depends only on personal utilities, the social welfare function is described as *utilitarian*

• Welfare function $W$ is general enough to include concern with producers

  – Companies – the producers of output – are owned by shareholders whose utility is contemplated in $W$ (.)

  – Thus, the “cost-side” of the economy is also included in $W$ by valuing the utilities/profits of shareholders

  – Beyond the impact of higher profits on the utility of a company’s shareholders, efficient production would be a constraint in the maximization of $W$ (.)

  – Because we are considering exogenous income processes, we do not have to worry about the previous item
Social Welfare and Pareto Improvements

- Social welfare function $W(\cdot)$ is such that Pareto improvements increase social welfare
  - That is, if the utility of a person $U_i$ increases and everyone else’s stays constant:
    $$\frac{\partial W}{\partial U_i} = \omega_i > 0, \quad i = 1, \ldots, n \quad (2)$$
  - The number $\omega_i$ is an individual’s social weight in the measurement of social welfare
  - The result in (2) says that everyone has positive weight in the measurement of society’s welfare
  - If any one person is better off and no one is worse off, society is better off

- Figure 7.3a shows a social welfare function for a society composed of two people
  - Social welfare is constant along the indifference curves $W_1$ and $W_2$, where $W_2$ indicates higher social welfare
  - Consistent with (2), Pareto-improving change from point 1 to 2 increases social welfare
  - However, a Pareto improvement is not necessary for social welfare to increase
  - A move from 1 to 3 still increases social welfare even though person 1 is now worse off than before
  - Society is also indifferent between points 1 and 4 even though person 1 is better off at 1 and person 2 is better off at 4
Interpersonal Comparisons of Utility

- In figure 7.3a, interpersonal comparisons of utility are being made
  - An interpersonal comparison of utilities is a judgement about comparative values of different people’s utilities
  - Interpersonal comparisons allow judgements about how changes in different people’s utilities affect social welfare
  - This is what allows us to draw indifference curves
- We are using a common utility function across people to compare the implications of changes in welfare in the population
  - We are also assuming that the measurement of utility is *cardinal*
  - That is, that the difference in utilities across two different situations has meaning (like differences in height or temperature)

- When the measurement of utility is ordinal, different utility numbers (over high and low income, for example) simply provide rankings of utility
  - With ordinal utility, we could not compare the consequences of distributing income in different ways among people
  - When Pareto-improvement takes place, however, we could nonetheless conclude that welfare had improved with ordinal utility
- Because the utility function is the same and utility is cardinal, we can compare the implications of taking income from person 1 and giving it to person 2
  - This income redistribution causes a reduction in the utility of person 1 and an increase in that of person 2
  - The cardinality of utility and using the same utility function makes these changes comparable
• The common utility function used rules out the possibility of a judgement that two people with the same income benefit differently or have different utilities

  – A common utility function preempts privilege or prejudice that would be present if some people were regarded as being capable of benefiting more from income than others

  – Of course it also prevents differencing across people in situations when that were adequate (think of a very ill person’s marginal utility of income...)

Anonymity

• Privilege or prejudice is also preempted by anonymity in the social welfare function

  – Figure 7.3b shows a social welfare contour that is symmetric around the 45° line

  – The symmetry ensures that the shape of the social contour is independent of the identities of the people whose utilities are measured on the axis

  – It does not matter, from the viewpoint of treatment by the social welfare function whether someone is (or will be) person 1 or person 2
Distribution of Predetermined Income

- The social welfare function can be used to describe a decision behind the veil of ignorance about how a predetermined amount of income $Y$ will be divided once people have emerged from behind the veil
  - Person 1 will receive income $y_1$ while person 2 will receive $y_2$ with
    \[ y_1 + y_2 = Y \]
  - The utilities of the two people from the division of $Y$ into $y_1$ and $y_2$ evaluated at the common utility function are
    \[ U_1 = U(y_1), \quad U_2 = U(y_2) \]
- Figure 7.4a shows the utility possibilities frontier
  - It maps all the utility possibilities for the two people in the economy from alternative distributions of $Y$

- All points on this line (all distributions of $Y$ among the two people) are Pareto efficient:
  - At any point along the frontier, no person can be made better off without making the other worse off
  - Movements along the frontier are not Pareto improving change: one person must be made necessarily worse off for the other to gain
- If all income is given to person 1, society is at point $S$; if $Y$ is in turn given to person 2, point $V$ is attained
  - Because utility functions are identical across individuals, these utilities are also identical: $OS = OV$
  - The remainder points on the frontier result from dividing income between the two people
• The concavity of the frontier is due to declining marginal utility from income

  - If society is at point $S$ (person 1 has all the income) and some income is given to person 2, person's 2 utility increases by more than person 1's declines

  - For example, the transfer of income to person 2 at $S$ moves society to point $F$ where person 2 has gained more than person 1 has lost

• Because of identical utility functions and diminishing marginal utility of income, the frontier is symmetrically concave around the 45° line

  - Maximal social welfare $W_2$ is achieved at point $E$ on the 45° line by equal division of income

  - It achieves ex-post equality of income and utility

The Leaky Bucket of Redistribution

• Social insurance – thus provided by the government – requires financing by compulsory taxation

  - Compulsory taxes introduce an excess burden of taxation because people can modify and adapt their behavior to avoid taxation

  - For this reason, redistributive taxation generally leads to efficiency losses referred to as the “leaky bucket” of redistribution

  - If income transfers are financed by governmental borrowing, the taxes and excess burdens will take place in the future

  - The leaky bucket would not be present if redistribution took place through voluntary payments to an insurance company

  - However, as discussed earlier, there is no scope for private insurance contracts once people emerge from the veil of ignorance
• The source of the leaky bucket is shown in figures 7.5a and 7.5b
  – In figure 7.5a, we see the labor supply schedule of person 1: for each wage, the number of hours that she is willing to work
  – At wage rate $w_1$, person 1 chooses to work $L_1$ hours
  – The area below the wage received and above the labor supply schedule measures the benefit for person 1 of working $L_1$ hours at wage $w_1$
• Then she faces a tax $t$ which reduces her net wage to $w_1 (1 - t)$
  – Person 1 finds it less interesting to work as many hours as before, substitutes leisure for work and reduces hours worked to $L_{11}$
  – The tax revenue paid to the government is given by the shaded rectangle in figure 7.5a, joining the points $w_1, B, D, w_1 (1 - t)$
  – The excess burden of taxation is the triangle $BCD$

• The excess burden measures the loss in welfare for person 1 that is not recovered by anyone else
  – The total welfare loss to person 1 is given by the area joining the points $w_1, C, D, w_1 (1 - t)$
  – The rectangle associated with the fiscal revenue is a subset of the total welfare loss and is transferred to the government
  – The excess burden triangle $BCD$ is not recovered by anyone else
• The tax revenue paid by person 1 is given as a subsidy to person 2
  – This subsidy may require person 2 not to work, which makes this person unproductive
  – Alternatively, the tax revenue can be used to subsidize person 2’s labor market wage
  – A subsidy of $s$ per hour of work raises person 2’s wage to $w_2 (1 + s)$
• Facing a higher wage, person 2 responds by increasing hours worked from $L_2$ to $L_22$
  – The income transfer received by 2 is shown by the rectangle in figure 7.5b and equals the tax revenue rectangle in figure 7.5a
  – However, the increase in person 2's surplus from labor market participation is only the area joining the points $w_2, J, Z, w_2(1 + s)$
  – The efficiency loss is the triangle $JVZ$ in figure 7.5b

• Persons 1 and 2 are making substitution decisions between work and leisure time
  – For this reason, the utility function now includes both consumption (income) and leisure
  – The substitution responses are the sources of the inefficiency losses

• Adding the two triangles representing the individual efficiency losses, we get the loss from the leak in the bucket of redistribution
  – These losses are personal
    – Person 1 suffers the loss $BCD$ as a taxpayer
    – Person 2 suffers the loss $JVZ$ as a reduction in the benefit of receiving transfers

• There are further leaks or losses because of the cost of tax administration and costs of taxpayers' compliance with tax laws
  – Additional costs come from public spending on social workers to certify a person's eligibility for the income transfers and who administer the income transfers to recipients
• When behind the veil of ignorance, people do not know if they will be lucky and experience $\mu_H$ or unlucky and receive $\mu_L$ instead
  
  – As a result, they do not know whether they will be taxpayers or transfer recipients
  
  – However, because they personally bear the efficiency losses described above, they take them into account when deciding how income should be redistributed after emerging from behind the veil

**Feasible Redistribution**

• The leaks in redistribution define feasible redistribution

  – Although people are anonymous behind the veil, we call them person 1 and person 2

  – In figure 7.6, points $F_1$ and $F_2$ on the utility possibilities frontier indicate alternative utilities obtained after income is realized, before taxes or subsidies

  – Outcome $F_1$ is favorable to person 1 and $F_2$ is favorable to person 2

  – The higher utility for person 1 at $F_1$ may reflect greater ability, more favorable family circumstances and/or better educational opportunities (the converse applies at $F_2$)
• Viewed from behind the veil of ignorance, $F_1$ and $F_2$ are equally likely and are symmetric:
  - $F_1$ is at the same distance from point $S$ as $F_2$ is from point $V$
  - Thus, $F_1$ and $F_2$ are also at the same distance from the equal utility point $E$ on the 45° line

• Behind the veil, people are identical and there is ex-ante equality
  - At either $F_1$ or $F_2$, people know who they are and there is inequality

• Because of the leaky bucket, point $E$ with social welfare $W_2$ is no longer feasible
  - Redistribution along the original utility possibilities frontier $SV$ could only take place if people’s supply of labor were independent of taxation

• The leaks from redistribution condition feasible redistribution to take place along either $F_1A$ or $F_2A$
  - Feasible redistribution is now along the constrained utility possibilities frontier beginning at $F_1$ or $F_2$, whatever the market outcome turns out to be
  - The excess burden of taxation increases with the tax revenue collected
  - This causes outcomes along $F_1A$ (or $F_2A$) to lie increasingly further inside $SV$ because of the greater tax revenue required to attain points closer to $A$
  - The line connecting points $F_1$, 1, $A$, 2 and $F_2$ is thus the new utility possibilities frontier once the leaks from redistribution are taken into account
• In figure 7.6, we see the maximization of social welfare subject to redistribution costs
  
  – If the outcome after income is realized (and thus emerging from behind the veil of ignorance) is $F_1$, social welfare is maximized at point 1.
  
  – If the market outcome turned out to be $F_2$, social welfare is maximized at point 2.
  
  – Despite its costs, income redistribution increases social welfare (in the picture).
  
  – Points $F_1$ and $F_2$ were on the indifference surface with social utility $W_0$ and redistribution allows a higher indifference curve to be attained, $W_1$.
  
  – Full redistribution would have led society to point $A$, but at $A$ social welfare is less than at either point 1 or 2.

3 Choice of Social Insurance Behind a Veil of Ignorance

• In figure 7.6, we saw the maximization of a social welfare function subject to the constraints of redistribution

  – The outcome specifies the social insurance contract to be implemented once people emerge from behind the veil of ignorance and incomes are realized.

  – In that particular case, the insurance contract would not deliver ex-post equality of incomes because of the redistribution costs.

  – The lucky person would still have higher income after taxation and redistribution than the person receiving the subsidy.
Choice of a social welfare function is therefore determinant in specifying the properties of the social insurance contract

- Many possibilities exist

- The limiting cases are associated with Jeremy Bentham and John Rawls, who lived centuries apart

**Bentham**

- English political economist Jeremy Bentham (1748-1832) proposed the objective of seeking “the greatest good for the greatest number”
  
  - This objective has been interpreted as implying that social welfare is defined by the sum of the utilities of the people in the population
  
  - For example, with \( n \) individuals in the economy, Bentham’s social welfare function is
    
    \[
    W = \sum_{i=1}^{n} U_i
    \]
    
    - This objective function gives all people equal social weight, equal to unity
    
    - Only the *sum* of utilities matters for the measurement of welfare here, the *distribution* is of no importance
For two people, Bentham’s social welfare function is

\[ W = U_1 + U_2 \]

- Indifference curves are straight lines with slope \(-1\) (figure 7.7a):

\[ dU = 0 = dU_1 + dU_2 \iff dU_1 = -dU_2 \]

Figure 7.7b shows the social insurance contract associated with Bentham’s social welfare function

- Points \(F_1\) and \(F_2\) are the equally likely market determined outcomes
- Feasible redistribution takes place along the lines \(F_1A\) or \(F_2A\)
- Beginning at either \(F_1\) or \(F_2\), income redistribution increases social welfare from \(W_0\) to \(W_1\)

Because utilities are not equalized at points 1 and 2,

*Bentham provides incomplete insurance*

Complete insurance would bring society to point \(A\) on the \(45^\circ\) line but, due to the costs of redistribution, welfare there is lower than at either 1 or 2
Rawls and the Weakest Link

- Philosopher John Rawls (1921-2002) proposed that maximizing social welfare requires maximizing the utility of the worst-off person in society:
  - If society is as strong as its weakest link, the worst-off person is the weakest link
  - Rawls proposed that, behind the veil of ignorance and not knowing who they will be in the future, people would wish all attention to be focused on the most unfortunate person whom they might be
  - After having done the best that is possible to help the worst-off person, society would then move on to the next worse off and so on

- Throughout this procedure, only the utility of the worst-off person ever matters for defining social welfare
  - Thus, Rawls social welfare function is of the max-min form:
    \[ W = \max \{ \min U \} \]
    - That is, social welfare is maximized when the utility of the person with the lowest utility is maximized

- Figure 7.8a shows the Rawls measure of social welfare
  - Because only improvements in the well-being of the poorest person count for social welfare, the Rawls social welfare function does not satisfy the condition that Pareto improvements increase social welfare
  - The right angles of the indifference curves at the 45° line indicate that there is no trade-off in social welfare between making richer and poorer better off
  - Only the welfare of the worst-off person counts in determining social welfare
• Figure 7.8b shows the social insurance contract emerging under Rawls’ social welfare function

  - Beginning from the market outcomes at either $F_1$ or $F_2$, where social welfare is $W_0$, social insurance requires redistribution along $F_1A$ or $F_2A$ until ex-post equality is achieved at $A$

  - Insurance at $A$ is complete because ex-post utilities after taxation and subsidies are equal

  - The social insurance contract increases social welfare from pre-redistribution $W_0$ to post-distribution $W_1$

• In the case shown in the picture, we see that

  *The Rawls social welfare function provides complete insurance*

• Although the Rawls social welfare function results in complete insurance and thereby in ex-post equality, Rawls’ objective function is not ex-post equality

  - Rather, ex-post equality is the consequence of the Rawls objective of maximization of the utility of the worst-off person

  - Because of the limitations on tax revenue as described by the Laffer curve, we will see that complete insurance and ex-post equality are not necessary consequences of the Rawls definition of social welfare

  - Nonetheless, we expect the social contract to provide more insurance and ex-post equality under Rawls than under Bentham

• Figure 7.9a shows how Bentham and Rawls are limiting cases of social welfare functions

  - We also see a symmetric social welfare function that is intermediate between the other two
Bentham and Rawls are limiting cases because Bentham cares only about the sum of utilities and not about their distribution

- Rawls cares only about the distribution of utilities and, in particular, about the utility of the worst-off person

How do we interpret the “intermediate” social welfare function?

- In figure 7.9b, we see an indifference curve of the intermediate function
- Along the indifference curves, the two people’s social weights change
- The two people only have equal weight at the 45° line, at point $H$ where their utilities are equal
- Away from the 45° line, the person with the lower utility has the greatest social weight

At a point such as $A$, the utility of person 1 exceeds largely that of person 2

- Here, the social indifference curve is much steeper than at the 45° line
- This means that

$$dU_1 = -\alpha dU_2,$$

where $\alpha >> 1$

- In order to keep social welfare constant, increments in $U_2$ are traded off against larger declines in $U_1$
- Society accepts a large reduction in $U_1$ in order to increment $U_2$ a little because $U_1$ is very high at $A$
• Suppose that people had chosen the social welfare function depicted in figure 7.9b behind the veil of ignorance

  Then, people would have agreed that social welfare would be constant when high-utility person 1 gives up utility $AB$ in order for person 2 to receive the smaller quantity $BD$

  The agreement behind the veil of ignorance would have specified that whoever turned out to have the lowest utility would have the highest social weight.

  The slope of the social utility function would also have specified social aversion to inequality.

  Social aversion to inequality is measured by the amount of utility that can be taken from a high-utility person and transferred to a low-utility one while keeping social welfare constant.

  In figure 7.9b, social aversion to inequality increases with departures in either direction from equality point $H$.

• Figure 7.10a compares Bentham and Rawls when there are no excess burdens of taxation and so no redistribution costs.

  Beginning at market outcomes $F_1$ or $F_2$, Bentham and Rawls both provide complete insurance at $E$, and so ex-post equality.

  This would also be the case under any symmetric social welfare function.

  Thus, without redistribution costs, aversion to inequality does not affect social insurance based on a symmetric social welfare function:

  Any symmetric social welfare function provides complete insurance and results in ex-post equality after income redistribution.
By contrast, figure 7.10b shows a very leaky redistribution bucket

- The feasible redistribution frontiers fall precipitously as redistribution takes place

- In the case depicted, no redistribution can improve on an agreement behind the veil of ignorance to remain at the market outcomes $F_1$ or $F_2$

- Under a Bentham social welfare function, no insurance is granted because the implementation costs outweigh the advantages

Under Rawls, however, social welfare is greater at complete ex-post equality point $A$ than at the market outcomes

- Under Rawls, therefore, society would see complete equalization of ex-post utilities and incomes

The comparison of figures 7.10a and 7.10b shows us that

Social insurance contracts associated with different social welfare functions differ because of different sensitivities of social welfare functions to the efficiency costs of redistribution

- Bentham is the most sensitive to efficiency costs whereas Rawls is not at all sensitive to those costs